Queuing theory as a basis for staff placement at a customs post

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Abstract. The paper presents the development of economic and mathematical tools to improve managerial decision-making for staff placement at the customs post. As an object of research, the authors analyze the performance of customs operations and customs control by customs officials in relation to individuals transporting goods for personal use across the customs border of the Eurasian Economic Union at the airport checkpoint – Samara Airport customs post. According to the logic of the queuing theory, a customs post official is a "service channel", and the flow of passengers at the customs control zone (arrivals area) is a "request". The optimal placement of officials required at the arrivals area was determined for each of the periods under consideration (12 months in 2019), as well as the average time spent by passengers in the customs control area was estimated. The justification of these parameters based on the queuing theory contributes to achieving the minimum cost of the staff of officials at Samara Airport customs post involved in dealing with individuals transporting goods for personal use across the EAEU customs border.
Keywords: staff placement, queuing theory, theory of queues, customs, customs post

1 Introduction

The strategic goal of the development of the Federal Customs Service of Russia is the formation of a qualitatively new, rich in "artificial intelligence", quickly reconfigured, information-related with internal and external partners, "smart" customs service, invisible to law-abiding business and effective for the state. One of the targets aimed at achieving the strategic goal is target 18 – improving the efficiency of personnel management of customs authorities which, among other things, includes improving the system of recruitment and staff placement [1, 2].

To solve the management problem of the staff placement at the customs post, the authors apply the queuing theory, which allows one to take into account the probabilistic

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nature of the processes of performing customs operations and conducting customs control in relation to individuals transporting goods for personal use across the customs border of the Eurasian Economic Union (hereinafter referred to as the EAEU).

2 Materials and methods

In English literature, queuing theory is often referred to as "the theory of -queues". This theory studies situations prevailing in a person's life where there is a limited resource and few customers who can use it, leading to service delays or failure of some customers. The desire to understand the objective reasons for these delays or failures and the possibility of reducing their impact is an incentive for the development of the theory of -queues [3-6].

In Russian literature, the term "queuing theory" belongs to A.Y. Khinchin. He outlined the basic concepts of the queuing theory in his monograph and is the creator of the foundations of queuing theory [3].

In general, the task of queuing systems (hereinafter referred to as QS) has the following content. Given: the average density (intensity) λ of the request flow; the average duration (intensity) μ of the request service; the costs associated with the maintenance of service channels during their downtime; the costs associated with the emergence of a queue of requests; the costs of maintaining service devices; the costs associated with the denial of service of requests when all channels are busy; and the number of active parallel channels n. It is necessary to determine the optimal number of service channels in relation to the selected criterion [3, 4, 7].

Performance indicators group for QS:

- absolute throughput (A) – average number of requests served per unit of time, or the intensity of the outgoing flow of requests served (this is a part of the intensity of the incoming flow of requests);
- relative throughput (Q) – ratio of absolute throughput to the average number of incoming requests per unit of time;
- average duration of the period when QS is busy (T_b);
- load intensity (ρ) shows the degree of consistency of the input and output flows of service channel requests and determines the QS stability;
- QS rate of use – average percentage of time when the system is busy with request servicing.

Request service quality indicators:

- average waiting time for the request in the queue (T_q);
- average residence time in QS (T_{ser});
- probability of denial-of-service requests without waiting time (p_{den});
- probability of immediate acceptance of the request (p_{acc});
- the distribution law of the request waiting time in the queue;
- average number of requests in the queue (N_q);
- average number of requests in QS (N_{ser});

Of all the variety of QSs encountered in practice, the authors consider the multi-channel QS with waiting time and unlimited queue to be the most applicable to the activities of officials at the customs post.

This QS is characterized by the following: a request received by the system at the time when all n channels are busy joins a queue and waits for its service. Any request received will be served.

QS can be in one of an infinite number of states:

S_k, $k = 0, 1, ..., n$, – k channels are busy and there is no queue;

S_m, $m = 0, 1, ...$, – all n channels are busy and there are m requests in the queue [1, 4, 16].

The study considers the QS where the service intensity of all n channels will be higher than the intensity of the incoming flow λ, otherwise, the queue of requests in the system will grow indefinitely over time, that is

$$\lambda < n\mu,\ or\ \psi < 1.$$

Formulas for the system state probabilities:

$$p_0 = \left(\sum_{k=0}^{n} \frac{n^k}{k!}\psi^k + \frac{n^n}{n!}\frac{\psi^{n+1}}{1-\psi}\right)^{-1}; \quad (1)$$

$$p_k = \{p_0 \frac{n^k}{k!}\psi^k,\ k = 1, 2, ..., n;\ p_0 \frac{n^n}{n!}\psi^k,\ k = n + 1, n + 2, ... \quad (2)$$

Probability of denial to accept the request

$$p_{den} = 0.$$

Probability of the incoming request acceptance

$$p_{acc} = 1.$$

Relative QS throughput

$$Q = 1.$$

Absolute QS throughput

$$A = \lambda.$$

Average number of busy channels (requests under service) is equal to the QS load indicator

$$\overline{N}_{ser} = \overline{K} = \frac{A}{\mu} = \rho. \quad (3)$$

Average number of requests in the system

$$\overline{N} = \overline{N}_{ser} + \overline{N}_q. \quad (4)$$

Average request service time

$$\overline{T}_{ser} = \frac{\overline{N}_{ser}}{\lambda} = \overline{K}/\lambda. \quad (5)$$

Average waiting time for a request in the service queue

$$\overline{T}_q = \frac{\overline{N}_q}{\lambda}. \quad (6)$$

Average request residence time in QS

$$\overline{T}_{aver} = \overline{T}_{ser} + \overline{T}_q = \frac{\overline{N}_{ser} + \overline{N}_q}{\lambda} = \frac{m}{\lambda} + 1/\mu. \quad (7)$$

The logic of the queuing theory includes the analysis of the time of passing the customs control by individuals, as well as performing customs operations at the airport checkpoint – Samara Airport customs post; a customs post official here is a service channel; and the flow of passengers passing through the customs control zone (arrival zone) is a request. Passengers arrive at the customs control zone, mostly not regularly, but at random times, so it should be noted that the service time is random.

Passenger traffic is characterized by the intensity of the frequency of occurrence of events or the average number of events entering the QS per unit of time, – λ. The random nature of the flow of passengers and the time of customs control and customs operations leads to an uneven load of the QS under consideration: in some periods of time, there is an overload in QS associated with a large passenger flow in the summer and on weekends; other time intervals are characterized by a small number of passengers that leads to QS underloading.

In the analyzed case, we consider the flow stationary, since its probabilistic characteristics do not depend on time; in particular, the intensity of the stationary flow is a constant value:

$$\lambda(t) = \frac{Q}{T}, \quad (8)$$

where $\lambda(t)$ – flow intensity, people/hour;

Q – passenger traffic, people;

T – the time a passenger passes through the customs control zone, *hour.*

Consider the process of customs control and performing customs operations in relation to passengers at the airport checkpoint – Samara Airport customs post – in order to describe and compare the QS characteristics under different number of channels. For this purpose, we selected and analyzed data on the statistics of monthly loading and the average intensity of the flow of requests per unit of time (hour) of the arrival area at Samara Airport customs post in 2019 (Table 1).

Table 1. Quantitative indicators of the arrival area at Samara Airport customs post in 2019

Period (2019)	**Number of incoming individuals at Samara Airport customs post, people**	**Average flow intensity of requests per unit of time λ, people/hour**
January	13 405	18
February	11 145	16.5
March	17 437	23.4
April	17 454	24.2
May	43 183	58
June	50 081	69.6
July	51 380	69

August	52 518	70.6
September	49 069	68.2
October	43 885	59
November	24 122	33.5
December	17 233	23.2

Source: Compiled by the authors

The motion and time study of customs operations and customs control in respect of goods for personal use transported by individuals across the EAEU customs border was carried out in the course of the study for three cases:

1. Accompanied baggage and hand luggage of individuals are not subject to customs inspection as a result of selective customs control in the "green" corridor of the arrival area.
2. Accompanied baggage and hand luggage of individuals are partially subject to customs inspection as a result of selective customs control in the "green" corridor of the arrival area.
3. Accompanied baggage and hand luggage of individuals are completely subject to customs inspection as a result of selective customs control in the "green" corridor of the arrival area.

If an individual passes through the "red" corridor of the arrival zone, the authors consider the maximum possible time for performing customs operations in respect of goods for personal use (one working hour). The statistics shows that the number of passenger customs declarations issued in 2019 at Samara Airport customs post amounted to 198 units, or 0.0087% of the total number of arriving passengers. Therefore, this case can be ignored when calculating the average time of customs operations, and, consequently, the loading of customs inspectors.

The average time of customs operations is calculated taking into account the obtained data of the conducted motion and time study. Out of 100% cases:

- on average, 20% are individuals whose accompanied baggage and hand luggage are not subject to customs inspection as a result of selective customs control in the "green" corridor of the arrival area;
- on average, 70% are individuals whose accompanied baggage and hand luggage are partially subject to customs inspection as a result of selective customs control in the" green" corridor of the arrival area;
- on average, 10% are individuals whose accompanied baggage and hand luggage are completely subjected to customs inspection as a result of selective customs control in the "green" corridor of the arrival area.

Therefore, let us assume T_{ser} in the generated model as average duration of customs control and customs operations with arriving passengers:

$$T_{ser} = 0.2 \cdot T_1 + 0.7 \cdot T_2 + 0.1 \cdot T_3 \quad (9)$$

$$T_{ser} = 0.2 \cdot 48 + 0.7 \cdot 82 + 0.1 \cdot 944 = 161.4\ (sec) = 0.045\ (hour). \quad (10)$$

3 Results

To solve the problems set above (determining the optimal number of service channels), it is necessary:

- to determine the minimum number of officials (customs inspectors) n_{min}, that could prevent the indefinite growth of the queue length, and the corresponding service characteristics at $n = n_{min}$;

- to determine the optimal number n_{opt} of customs inspectors, which ensures the minimum in the relative cost C_{rel} associated with the cost of maintaining service channels and minimum waiting time for passengers in the queue;
- to compare service characteristics at $n = n_{min}$ and $n = n_{opt}$.

A detailed calculation is provided for the January 2019 data.

By condition $\lambda = 18$ people/hour.

Then, the given channel load intensity which expresses the average number of requests received during the average service time of one request:

$$\rho = \frac{\lambda}{\mu}, \tag{11}$$

where $\mu = \frac{1}{T_{insp}}$ – average service duration by a customs inspector for one passenger.

$$\mu = \frac{1}{0.045} = 22.22.$$
$$\rho = \frac{18}{22.22} = 0.82.$$

The given channel load intensity expresses the average number of requests received over the average service time of one request.

The queue will not grow indefinitely if

$$\frac{\rho}{n} < 1,$$

i. e. at $n > \rho = 0.82$.

Therefore, the minimum number of customs inspectors in January 2019 is $n_{min} = 1$.

The number of inspectors in February – December 2019 is determined similarly (Table 2).

Table 2. Minimum number of inspectors calculated according to the queuing theory (2019), *people*.

Month	**01**	**02**	**03**	**04**	**05**	**06**	**07**	**08**	**09**	**10**	**11**	**12**
n_{min}	1	1	2	2	3	4	4	4	4	3	2	2

Source: Compiled by the authors

Further, the characteristics of the QS service for each of the considered periods (monthly in 2019) are defined and shown in Table 3.

Table 3. Service characteristics of a multi-channel QS with an infinite queue

№	**Service characteristics**	**Period (month 2019 г.)**											
		01	**02**	**03**	**04**	**05**	**06**	**07**	**08**	**09**	**10**	**11**	**12**
1	Minimum number of channels	1	1	2	2	3	4	4	4	4	3	2	2
2	Request flow intensity	18	16.5	23.4	24.2	58	69.6	69	70.6	68.2	59	33.5	23.2
3	Service time	0.045											
4	Service flow intensity	22.22											
5	Load indicator	0.81	0.743	1.053	1.089	2.61	3.132	3.105	3.177	3.069	2.655	1.508	1.044

6	Load indicator per channel	0.81	0.743	0.527	0.545	0.87	0.783	0.776	0.794	0.767	0.885	0.754	0.522
7	Probability of downtime	0.552	0.574	0.384	0.373	0.1	0.055	0.056	0.053	0.058	0.097	0.274	0.386
8	Probability of failure	0											
9	Probability of maintenance	1											
10	Relative throughput	1											
11	Absolute throughput	18	16.5	23.4	24.2	58	69.6	69	70.6	68.2	59	33.5	23.2
12	Average number of busy channels	0.81	0.743	1.053	1.089	2.61	3.132	3.105	3.177	3.069	2.655	1.508	1.044
13	Average number of requests in the queue	10.032	4.798	0.5	0.58	15.255	3.667	3.363	4.221	3.036	20.247	3.882	0.481
14	Average number of requests in the system	10.842	5.541	1.553	1.669	17.865	6.799	6.468	7.398	6.105	22.902	5.39	1.525
15	Average time for request service	0.045											
16	Average waiting time in the service queue	0.557	0.291	0.021	0.024	0.263	0.053	0.049	0.06	0.045	0.343	0.116	0.021
17	Average time of request residence in QS	0.602	0.336	0.066	0.069	0.308	0.098	0.094	0.105	0.09	0.388	0.161	0.066

Source: Compiled by the authors

In this paper, the cost-effectiveness (minimum costs for actions mentioned above) is a key factor in evaluating the effectiveness of proposals to optimize the staff placement at Samara Airport customs post and the time of customs control in respect of goods for personal use transported by individuals across the EAEU customs border.

To assess the cost effectiveness of the staff at Samara Airport customs post involved in dealing with individuals transporting goods for personal use across the EAEU customs border, the relative cost value is calculated for other values *n* (the number of officials):

$$C_{rel} = \frac{n}{\lambda} + 3 \cdot \overline{T}_q. \quad (12)$$

Table 4 shows relative cost values obtained for the period under consideration (January 2019).

Table 4. Relative value of costs for various values of the number of officials in January 2019

Service Characteristics	Number of officials				
	1	**2**	**3**	**4**	**5**
Probability of downtime	0.552	0.468	0.449	0.446	0.445

Average number of requests in the queue	10.032	0.176	0.02	0.003	0
Relative value of costs	1.862	0.276	0.305	0.357	0.413

Source: Compiled by the authors

Therefore, the optimal number of officials in January 2019 is $n = n_{opt} = 2$, since the relative cost is minimal (0.276) at this value.

Compared to the actual number of officials in the arrival area at the same time (4 people), the relative cost is reduced by 22.7%.

Similar calculations for the remaining periods (February – December 2019) are presented in Table 5.

Table 5. Optimal number of officials at Samara Airport Customs post in the arrival area in 2019

Period	**01**	**02**	**03**	**04**	**05**	**06**	**07**	**08**	**09**	**10**	**11**	**12**
n_{opt}	2	2	3	3	5	6	6	6	5	5	3	3
n_{act}	4											
Δn	-2	-2	-1	-1	+1	+2	+2	+2	+1	+1	-1	-1
ΔC_{rel}	22.7	25.73	12.09	10.67	7.63	35.33	32.94	39.25	30.4	9.84	3.11	12.05

Source: Compiled by the authors

Table 6 shows a similar estimate of the time of stay of passengers when the optimal number of officials are involved throughout the analyzed period.

Table 6. Estimation of the average time of stay of passengers at $n = n_{opt}$, *sec.*

Period	**01**	**02**	**03**	**04**	**05**	**06**	**07**	**08**	**09**	**10**	**11**	**12**
$\overline{T}_q$	198	191	169	173	173	169	169	169	187	173	191	169
$\overline{T}_{act}$	162											
$\Delta\overline{T}$	-36	-29	-7	-11	-11	-7	-7	-7	-25	-11	-29	-7

Source: Compiled by the authors

In the period under consideration (January, 2019), the average time of stay of passengers in the customs control zone in case of acceptance of the proposals developed above, would be 0.055 hour (198 seconds).

Over the analyzed period the stay of passengers in the customs control zone will slightly increase (7-36 seconds), taking into account the calculated staff placement at the customs post.

4 Discussion

Analysis of the existing scientific background on the topic under consideration shows that the use of mathematical tools to solve the problem of improving the activities of customs posts, other state structures and security services is actively applied by many researchers in different countries [8-16]. Thus, the authors of the work [9] identify the problems that affect the managerial success of airport security services in Turkey and develop solutions to these problems.

This work also emphasizes the importance of the study conducted, since one of the main professional tasks of customs post officials is to prevent attempts to illegally transport narcotics, psychotropic, potent substances and their precursors, weapons, ammunition, cultural, archaeological and other valuables, as well as other prohibited goods across the

EAEU customs border. The optimal number of customs post officials determined using the mathematical apparatus will contribute to the most successful solution of this problem.

The paper [8] considers the process of performing customs operations and customs control of individuals on road transport in the form of a queuing system, which allows the authors to present options for passenger service with a given number of customs inspectors and optimize the time and costs of individuals when passing customs control at the seaport border checkpoint.

5 Conclusion

The key factor in evaluating the effectiveness of proposals to optimize the staff placement of officials at Samara Airport customs post and the time of customs control in respect of goods for personal use transported by individuals across the EAEU customs border was identified as cost effectiveness (minimum costs for these measures). Thus, for each of the periods under consideration, the optimal number of officials required in the working process in the arrival area at Samara Airport customs post was calculated, and the average time of stay of passengers in the customs control area was estimated.

The results obtained in the study are universal and can be used as an economic and mathematical tool for justifying the staff placement of officials at other customs posts.

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